Maths Skills for Public Health

Working With Decimal Numbers

These notes are designed to help you understand and use some of the mathematical tools that will arise during your studies.

You are welcome to visit the Maths Learning Centre in person whenever you feel the need. Our Drop-In Centre in Hub Central is open **10.00am** to **4.00pm** Monday to Friday during teaching periods, mid-semester breaks and exams (location and contact details are on the next page).

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Fractions, Decimals and Percentages

A fraction like $\frac{4}{50}$ can be thought of as \4 out of 50". Doubling both gures produces the fraction $\frac{8}{100}$ or \8 out of 100". It's easy to see that they both represent the same thing.

The word *percent* means \per hundred" or \out of 100" so both versions of this fraction represent \8%". In fact, so do all of these fractions too:

$$\frac{4}{50}, \quad \frac{8}{100}, \quad \frac{2}{25}, \quad \frac{40}{500}, \quad \frac{800}{10{,}000}, \quad \text{etc.}$$

What if the fraction is more complicated, such as $\frac{5}{8}$ (\5 out of 8")? If you can't see a simple way to create a bottom line of 100, convert the fraction to a *decimal* rst. This can be done on a calculator (or, if you like that sort of thing, long division).

$$\frac{5}{8} = 0.625.$$

If you are unsure about what this means, the various digits are described here:

The decimal 0.625 is read \zero point six two ve" and consists of 6 tenths, 2 hundredths and 5 thousandths. In other words, it's bigger than a half and smaller than one.

The decimal form can be thought of as a number \out of 1" so, to convert this to a percentage (\out of 100"), simply multiply it by 100. If you try this on a calculator you will get

$$62.5\%$$

Notice that the digits remain the same but the decimal point has moved two places to the right. This is one of the advantages of our \base ten" number system (ie. ten digits). The topic \Working With Decimals"discusses this concept further.

Example: Convert $\frac{1}{3}$ to a percentage.

On a calculator we get

$$\frac{1}{3} = 0.333\ldots,$$

which is known as a *recurring decimal*. Multiplying by 100 we get

$$\frac{1}{3} = 0.333\ldots \text{ or } 33.3\ldots\%$$

Example: On Census night 2001, there were 18,972,350 people in Australia (including overseas visitors) compared with 17,892,423 in 1996. By what percentage did the population grow in this time?

Taking the ratio of 2001 to 1996 we get

$$\frac{18,972,350}{17,892,423} = 1.06035666\ldots$$

from the calculator. This "top heavy" fraction represents a number larger than 1 as you can see from the number (1) in the units position of the decimal form.

Multiplying by 100 we see that the size of the population in 2001 was $106.035666\ldots\%$ of its size in 1996. This says that the population of Australia in 2001 was a bit over 106% of the population size in 1996. In other words, the population grew by a bit over 6% between 1996 and 2001.

Exercise 1

Convert the following fractions to percentages:

(a) $\frac{19}{100}$ (b) $\frac{1}{4}$ (c) $\frac{3}{5}$ (d) $\frac{8}{4}$

The Power of Tens

Numbers that we work with can be become very large. For example, there were 4,647 people in Australia aged 95 and over at the 2001 Census. In words, we might say \over four and a half thousand" so that the number is easier to describe. This changes the **unit of measurement** from *people* to *thousands of people* .

A useful feature of our \base 10" number system is that, if we change the units of a number to a **power of 10** (eg. hundreds, thousands, tens of thousands, etc), the actual digits remain the same and the decimal point \moves" to the left a number of \steps" equal to the power of 10 involved.

Example: \One thousand" can be written as 1;000 or 10^3 (\a one with **three** zeros after it"). Hence

4;647 = 4:647 thousand.

The decimal point (assumed to be after the 7) has moved back 3 steps.

Example: There were 18,769,249 people in Australia at the 2001 Census, meaning \more than 18 and a half million":

18;769;249 = 18:769249 million.

Since \one million" is 1;000;000 or 10^6, the decimal point has moved back 6 steps.

Exercise 2

Express the following numbers in the units given and state the power of 10 involved:

(a) 10,312 in thousands

(b) 9,502,703 in millions

(c) 1,027,015,247 in millions

(d) 1,027,015,247 in billions (1 billion = 1,000,000,000)

Exponential/Scientific Notation?

If we use powers of 10 to "move" the decimal point in large numbers to sit next to the *first* digit, the number is said to be in **Exponential** or **Scientific Notation**:

$$4{,}647 = 4.647 \times 1{,}000 = 4.647 \times 10^3$$

$$18{,}769{,}249 = 1.8769249 \times 10{,}000{,}000 = 1.8769249 \times 10^7$$

In this form, the power of 10 gives an indication of the size or *magnitude* of the number.

The same can be done for very small numbers such as 0.0018, by moving the decimal point the other way and using *negative* powers of ten:

$$0.0018 = 1.8 \text{ thousandths} = 1.8 \times \frac{1}{1{,}000} = 1.8 \times 10^{-3}$$

$$0.00000345 = 3.45 \text{ millionths} = 3.45 \times \frac{1}{1{,}000{,}000} = 3.45 \times 10^{-6}$$

If you are unsure about the decimal positions used above (thousandths, millionths, etc) see **Appendix A**.

In the following exercises it may help to use the table of powers of ten and the various ways of writing them in **Appendix A**.

Exercise 3

Convert the following numbers to Scientific Notation:

(a) 38,000 (b) 0.04 (c) 0.000019

(d) 0.010004 (e) 710.5 (f) 1,963.09

Convert the following numbers to ordinary decimals:

(g) 2.65×10^3 (h) 1.57×10^{-4} (i) 1.5×10^8

(j) 5.005×10^{-2}

Rates

If you drive 180 kilometres in 3 hours, your average speed was

$$\frac{180\text{ km}}{3\text{ hours}}$$ = 60 \km per hour" (or \km/h").

Scientific/Exponential Notation on a Calculator or Computer

If a calculation produces a number too large or too small for your scientific or graphic calculator to display directly, it will switch to scientific/exponential notation.

If you want to enter a number in scientific/exponential notation, the [EXP] key is a good shortcut. For example, to enter $1:2 \quad 10^5$, type

[1] [ ] [2] [EXP] [5]

Example: To enter $5:34 \quad 10^{\ 6}$, type

[5] [ ] [3]

Rounding Numbers

Calculations often produce answers with many decimal places. For example, \2 people out of every 3" represents an *exact* percentage of 66:6666 : : :% but, for simplicity we would probably round this o to the rst decimal place or the nearest whole number.

The process of rounding numbers is as follows:

(i) Decide where you want your rounded value to stop.

(ii) Look at the *next digit* on the right. If it is a

5, 6, 7, 8 or 9 then increase the *last digit of the rounded value* by 1.

0, 1, 2, 3 or 4 then do nothing.

To round 66:6666 : : : to one decimal place, we look at the *second* decimal place. Since this is a 6 we round up the rst decimal place: Answer: 66.7%.

Examples:

(1) [31)[31)er:[31)r7(place)326(oer:)-435(66.7%.)]TJ/F45 s.nd

Exercise 5

Round:

(a) 3.812 to two decimal places

(b) 4.56 to one decimal place

(c) 105.5 to the nearest whole number

(d) 0.0034 to two decimal places

(e) 15.0999 to one decimal place

(f) 95.4999 to the nearest whole number

(g) 95.9999 to three decimal places

(h) 0:00444 : : : to three decimal places

Appendix A: Powers of 10 and Decimal Places

	⋮		
millions	$1;000;000$	10^6	
hundred thousands	$100;000$	10^5	
ten thousands	$10;000$	10^4	
thousands	$1;000$	10^3	
hundreds	100	10^2	
tens	10	10^1	(anything to the power 1 is itself)
ones	**1**	10^0	(anything to the power 0 is 1)
tenths	$0:1$	$10^{\ 1}$	$\frac{1}{10}$
hundredths	$0:01$	$10^{\ 2}$	$\frac{1}{100}$
thousandths	$0:001$	$10^{\ 3}$	$\frac{1}{1;000}$
ten thousandths	$0:0001$	$10^{\ 4}$	$\frac{1}{10;000}$
hundred thousandths	$0:00001$	$10^{\ 5}$	$\frac{1}{100;000}$
millionths	$0:000001$	$10^{\ 6}$	$\frac{1}{1;000;000}$
	⋮		

Example of decimal places:

Appendix B: Short Answers to Exercises

Answers 1

(a) 19% (b) 25% (c) 60% (d) 88.88...%

(e) 99.0909...% (f) 125% (g) 100.9090...% (h) 0.5% (ie. “half of 1%”)

(i) 72.49...%

(j) $0.671 \times 389 = 261.019$ so there were 261 deaths. (The answer was not a whole number because the quoted “67.1%” was rounded off. See “Working With Decimal Numbers: Rounding Numbers”.

Answers 2

(a) 10.312 thousand ($1{,}000 = 10^3$) (b) 9.502703 million ($1{,}000{,}000 = 10^6$)

(c) 102.7015247 million (d) 1.027015247 billion ($1{,}000{,}000{,}000 = 10^9$)

The number in (b) is the number of women in Australia according to the 2001 Census.

The number in (c) and (d) is the provisional population of India according to their 2001 Census. It meant that India officially became the second country in the world after China to exceed one billion people.

Answers 3

(a) 3.8×10^4 (b) 4×10^{-2} (c) 1.9×10^{-5}

(d) 1.0004×10^{-2} (e) 7.105×10^2 (f) 1.96309×10^3

(g) 2,650 (h) 0.000157 (i) 150,000,000

(j) 0.05005

Answers 4

Crude “death rate per woman” = 0.0064938...

(a) 649.38... deaths per 100,000 women (b) 6.4938... deaths per 1,000 women
Note: Normally we would round off to say “649.4”. See **Tip: Rounding Numbers** for more details.

(c) 9.6 cases per 100,000 people

Answers 5

(a)	3.81	(b)	4.6
(c)	106	(d)	0.00
(e)	15.1	(f)	95
(g)	96.000	(h)	0.004

Appendix C: Detailed Answers to Exercises

Detailed Answers 1

Convert the following fractions to percentages:

(a) $\frac{19}{100}$

= “19 out of 100” or 19%

(b) $\frac{1}{4} = \frac{25}{100}$

= “25 out of 100” or 25%

(or type “1 ÷ 4” into a calculator to give 0.25)

(c) $\frac{3}{5} = \frac{60}{100} = 60\%$

(or “3 ÷ 5” on a calculator gives 0.6)

(d) $\frac{8}{9}$

= $0.888\ldots$ on a calculator

= $88.8\ldots\%$

(e) $\frac{109}{110}$

= $0.990909\ldots$ on a calculator

= $99.0909\ldots\%$

(f) $\frac{5}{4}$

= $\frac{125}{100}$ or 125%

(or “5 ÷ 4” on a calculator gives 1.25)

(g) $\frac{111}{110}$

= $1.009090\ldots$ on a calculator

= $100.9090\ldots\%$

(h) $\frac{1}{200}$

= 0.005 on a calculator

= 0.5% (ie. “half of 1%”)

389 Adelaide residents aged 15 to 24 died between 1996 and 1999. Of these, 282 were male.

(i) What percentage of these deaths were male?

$$\frac{282 \text{ males}}{389 \text{ deaths}} = 0.72493\ldots = 72.49\ldots\%$$

67.1% of these 389 deaths were caused by injury or poisoning.

(j)$^{?}$ How many actual deaths does this percentage represent?

67.1% of $389 = 0.671 \times 389 = 261.019$, so there were 261 deaths. (The answer was not a whole number because the quoted “67.1%” was rounded off. See “Rounding Numbers”.)

Detailed Answers 2

Express the following numbers in the units given and state the power of 10 involved:

(a) 10,312 in thousands

Power of 10 used is 3 so move the decimal point back 3 places:
10.312 thousand

(b) 9,502,703 in millions

Power of 10 used is 6 so move the decimal point back 6 places:
9.502703 million

(c) 1,027,015,247 in millions

Power of 10 used is 6 so move the decimal point back 6 places:
102.7015247 million

(d) 1,027,015,247 in billions (1 billion = 1,000,000,000)

Power of 10 used is 9 so move the decimal point back 9 places:
1.027015247 billion

Detailed Answers 3

Convert the following numbers to Scientific Notation:

(a)	$38{,}000$	(b)	0.04	(c)	0.000019
	$= 3.8 \times 10^{4}$		$= 4 \times 10^{-2}$		$= 1.9 \times 10^{-5}$
(d)	0.010004	(e)	710.5	(f)	$1{,}963.09$

Detailed Answers 4

There were 9,502,703 women in Australia on Census Day 2001 and 61,709 female deaths registered in the same year. Work out the crude "death rate per woman"

$$\frac{61{,}709}{9{,}502{,}703} = 0.0064938\ldots$$

and convert it to:

(a) deaths per 100,000 women

$$\begin{aligned} & 0.0064938\ldots \times 100{,}000 \\ = {} & 0.0064938\ldots \times 10^5 \\ = {} & 649.38\ldots \text{ deaths per 100,000 women} \end{aligned}$$

(b) deaths per 1,000 women

$$\begin{aligned} & 0.0064938\ldots \times 1{,}000 \\ = {} & 0.0064938\ldots \times 10^3 \\ = {} & 6.4938\ldots \text{ deaths per 1,000 women} \end{aligned}$$

In a hypothetical area, 12 cases of a particular syndrome were reported last year out of a population of 125,000. Work out:

(c) the incidence rate per 100,000 people.

The incidence rate per person is $\frac{12}{125{,}000} = 0.000096\ldots$, so the rate per 100,000 people is

$$\begin{aligned} & 0.000096\ldots \times 100{,}000 \\ = {} & 0.000096\ldots \times 10^5. \\ = {} & 9.6. \end{aligned}$$

Note: It's a good idea to check if your answer looks reasonable. In this case, 12 out of 125,000 is roughly 10 out of 100,000.