Maths Learning Service: Revision

Matrices

Intro. to Fin. Maths I

When giving matrices a name, use capital letters such as A, B, etc to distinguish them from algebraic scalars such as a, b, etc.

Exercises

(1) Given that

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 4 & 5 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 7 & 1 & -3 \\ 2 & 0 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 \\ -4 & 9 \end{pmatrix} \quad D = \begin{pmatrix} 11 & 5 \\ 0 & -2 \end{pmatrix} \quad E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 3 \end{pmatrix}$$

find the following (if possible):

(a) $A + B$ (b) $B + A$ (c) $C + D$ (d) $C - D$ (e) $D - C$

(f) $A + E$ (g) $B - D$ (h) $3A$ (i) $2C + D$ (j) $5B - 4E$

Matrix Multiplication

The rule for multiplying matrices can be represented as follows:

$$AB = \begin{pmatrix} \text{row 1 of } A \times \text{col 1 of } B & \text{row 1 of } A \times \text{col 2 of } B & \text{row 1 of } A \times \text{col 3 of } B & \dots \\ \text{row 2 of } A \times \text{col 1 of } B & \text{row 2 of } A \times \text{col 2 of } B & \text{row 2 of } A \times \text{col 3 of } B & \dots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

where "row i of $A \times$ col j of B" is a single number and stands for "each entry in row i of A is multiplied by the corresponding entry in column j of B and the results are added together".

Examples:

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 4 & -5 \\ -1 & -2 \\ 0 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(i) $$CA = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 2 + 2 \times 1 & 1 \times (-1) + 2 \times 4 & 1 \times 3 + 2 \times 5 \\ 3 \times 2 + 4 \times 1 & 3 \times (-1) + 4 \times 4 & 3 \times 3 + 4 \times 5 \end{pmatrix} = \begin{pmatrix} 4 & 7 & 13 \\ 10 & 13 & 29 \end{pmatrix}$$

(ii) $$AB = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 4 & -5 \\ -1 & -2 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \times 4 + (-1) \times (-1) + 3 \times 0 & 2 \times (-5) + (-1) \times (-2) + 3 \times 3 \\ 1 \times 4 + \quad 4 \times (-1) + 5 \times 0 & 1 \times (-5) + \quad 4 \times (- \end{pmatrix}$$

> *Matrix multiplication is only defined when the number of columns in the first matrix equals the number of rows in the second.*

(iv) $CD = \begin{pmatrix} 13 & 19 \\ 27 & 43 \end{pmatrix}$ but $DC = \begin{pmatrix} 16 & 22 \\ 27 & 40 \end{pmatrix}$ so $CD \neq DC$.

> *In general $AB \neq BA$ for matrices.*

(v) $$CI = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1\times1+2\times0 & 1\times0+2\times1 \\ 3\times1+4\times0 & 3\times0+4\times1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = C \text{ (unchanged)}$$

(vi) $$IC = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = C \text{ (unchanged)}$$

> *The matrix I is an identity matrix and is the matrix equivalent of the number 1 in scalar multiplication.*

Notes:

1. The identity is an exception to the general rule for matrix multiplication since $CI = IC = C$.
2. Identity matrices only exist for square matrices. The matrix I used in Examples (v) and (vi) is called "the identity matrix for a 2×2 matrix". The identity matrix for a 3×3 matrix is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Exercises

$A = \begin{pmatrix} 1 & 0 \\ -1 & -2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$ $C = \begin{pmatrix} -1 \\ \end{pmatrix}$

Answers to Exercises

(1) (a) $\begin{pmatrix} 6 & 3 & -3 \\ 6 & 5 & 9 \end{pmatrix}$ (b) same as (a) (c) $\begin{pmatrix} 12 & 7 \\ -4 & 7 \end{pmatrix}$ (d) $\begin{pmatrix} -10 & -3 \\ -4 & 11 \end{pmatrix}$

(e) $\begin{pmatrix} 10 & 3 \\ 4 & -11 \end{pmatrix}$ (f) not possible (g) not possible (h) $\begin{pmatrix} -3 & 6 & 0 \\ 12 & 15 & 9 \end{pmatrix}$

(i) $\begin{pmatrix} 13 & 9 \\ -8 & 16 \end{pmatrix}$ (j) not possible

(2) (a) $\begin{pmatrix} 2 & 3 & 1 \\ -8 & -5 & -5 \end{pmatrix}$ (b) not possible (c) D (d) D

(e) not possible (f) $\begin{pmatrix} 3 \\ 6 \\ 15 \end{pmatrix}$ (g) $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ (h) not possible

(i) $E^2 = EE = \begin{pmatrix} 11 & 8 \\ 4 & 51 \end{pmatrix}$ (j) not possible