Maths Learning Service: Revision

Basic Trigonometry and Radians

Mathematics IA

Mathematics IMA

Pythagoras' Theorem

Other commonly used angles are easily converted as well:

$$\frac{\pi^c}{2} = 90^\circ, \quad \frac{\pi^c}{4} = 45^\circ, \quad \frac{\pi^c}{3} = 60^\circ, \quad \frac{\pi^c}{6} = 30^\circ.$$

From above, a useful conversion factor is

$$\frac{\pi^c}{180^\circ} \text{ for degrees to radians} \quad \left(\text{or } \frac{180^\circ}{\pi^c} \text{ for radians to degrees}\right).$$

Examples:

(i) $\dfrac{\pi^c}{4} = \dfrac{180^\circ}{4} = 45^\circ$

(ii) $3\frac{1}{3}\pi^c = \dfrac{10}{3} \times 180^\circ = 600^\circ$

(iii) $60^\circ = \dfrac{60^\circ}{1} \times \dfrac{\pi^c}{180^\circ} = \dfrac{\pi^c}{3}$

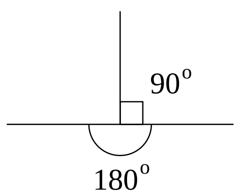
(iv) $720^\circ = \dfrac{720^\circ}{1} \times \dfrac{\pi^c}{180^\circ} = 4\pi^c$

(v) $2^c = \dfrac{2^c}{1} \times \dfrac{180^\circ}{\pi^c} = 114.59^\circ$

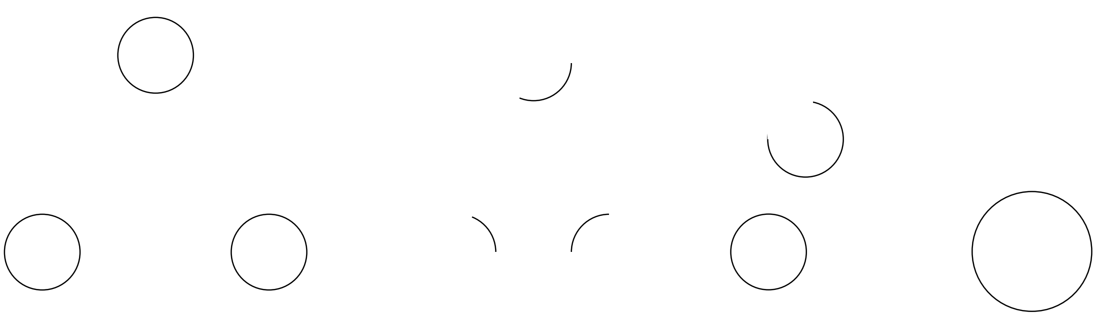
Your calculator should have a DRG or Mode button. This allows you to tell the calculator which units you are going to use for angles.

Triangles

In any triangle the angles add to 180°. (The angles in a straight line also add to 180°.)



The three types of triangle are:



$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{opposite}}{\text{hypotenuse}} \div \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{opposite}}{\text{adjacent}}.$$

Using the first letter of each definition and the sides involved produces the 'word'

"SohCahToa",

which may help you remember them.

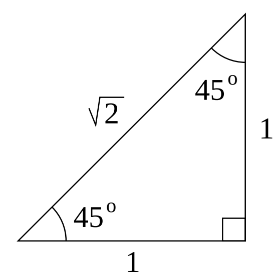
The following two simple triangles allow us to work out these trigonometric ratios *exactly* for certain angles.

1. (First check that the side lengths satisfy Pythagoras' theorem.)

$$\sin 45^\circ = \sin\frac{\pi^c}{4} = \frac{1}{\sqrt{2}} = 0.707 \text{ (to 3 dec. pl.)}$$

$$\cos 45^\circ = \sin\frac{\pi^c}{4} = \frac{1}{\sqrt{2}} = 0.707$$

$$\tan 45^\circ = \tan\frac{\pi}{4} = \frac{1}{1} = 1.$$



2. (Again, check that the lengths of the sides of the right-angled triangles satisfy Pythagoras' theorem.)

$$\sin 30^\circ = \sin\frac{\pi^c}{6} = \frac{1}{2}$$

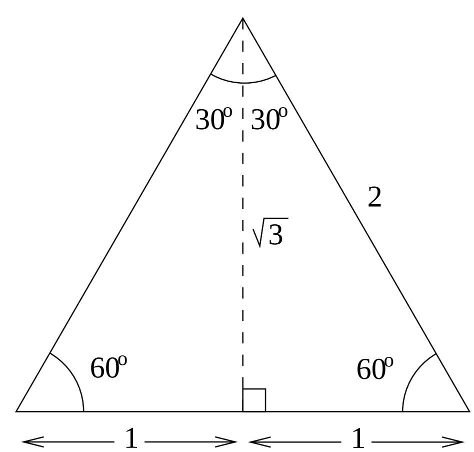
$$\cos 30^\circ = \cos\frac{\pi^c}{6} = \frac{\sqrt{3}}{2} = 0.866 \text{ (to 3 d.p.)}$$

$$\tan 30^\circ = \tan\frac{\pi^c}{6} = \frac{1}{\sqrt{3}} = 0.577 \text{ (to 3 d.p.)}$$

$$\sin 60^\circ = \sin\frac{\pi^c}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \cos\frac{\pi^c}{3} = \frac{1}{2}$$

$$\tan 60^\circ = \tan\frac{\pi^c}{3} = \sqrt{3} = 1.732 \text{ (to 3 d.p.)}.$$



For other angles, we usually use a calculator to find sines, cosines and tangents, for example

$$\tan 50^\circ = 1.192$$

$$\cos 1.1 = 0.454$$

If the degree symbol ° is not present, it is assumed that the angle is measured in radians.

Check these results on your calculator and remember to use the $\boxed{\text{DRG}}$ or $\boxed{\text{Mode}}$ button ct98w22roecres.

Problems Using Trigonometry

Example: For a person viewing a tall building, the angle of elevation of the top of the building is 49°. If the building is 100 m tall, how far is the man from the building?

Solution: Let the distance be d m. Then

$$\frac{100}{d} = \tan 49^\circ$$

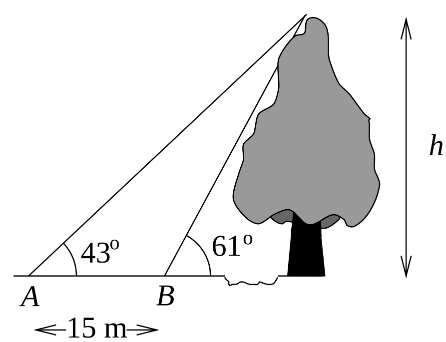
$$\therefore \quad d = \frac{100}{\tan 49^\circ} = 86.9 \text{ m.}$$

Exercises

(12) A small boat 500 m out to sea observes a cliff top on the shore at an angle of elevation of 21°. What is the height of the cliff?

(13) A surveyor wishes to establish the distance x from point A to point B on the other side of a lake. The distance AC in the diagram is measured to be 275 m and the angle $\angle BAC$ is 40°. Find x.

(14) [A bit harder] A forestry worker wishing to establish the height h of a tall tree on the other side of a river bank measures the angle of elevation of the top of the tree to be 61° at A, but 43° at a point B 15 m from A. Find the height of the tree.



Notice from the exercises that trigonometry is often useful in situations where a direct)efoin3386.6860Td[(