



Index laws are the rules for simplifying expressions involving *powers* of the same *base* number.

$$a^m \times a^n = a^{m+n} \quad \text{First Index Law}$$

$$(a^m)^n = a^{mn} \quad \text{Second Index Law}$$

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{Third Index Law}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^0 = 1$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Examples: Simplify the following expressions, leaving only positive indices in the answer.

$$\begin{aligned} \text{(a)} \quad & \frac{3^6 2^4}{3^4} \\ &= \frac{3^6}{3^4} \times 2^4 \\ &= 3^2 2^4 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 3^2 \times 3^{-5} \\ &= 3^{-3} \\ &= \frac{1}{3^3} \\ &= \frac{1}{27} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \frac{9(x^2)^3}{3xy^2} \\ &= \frac{9}{3} \times \frac{x^6}{x} \times \frac{1}{y^2} \\ &= 3 \times x^5 \times \frac{1}{y^2} \\ &= \frac{3x^5}{y^2} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & a^{-1} \sqrt{a} \\ &= a^{-1} a^{\frac{1}{2}} \\ &= a^{-\frac{1}{2}} \\ &= \frac{1}{a^{\frac{1}{2}}} \text{ or } \frac{1}{\sqrt{a}} \end{aligned}$$

Notes: (1) More involved fractional powers can be dealt with by noting that $a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m$ by the Second Index Law. For example,

$$(27)^{\frac{2}{3}} = (27^{\frac{1}{3}})^2 = (\sqrt[3]{27})^2 = (3)^2 = 9.$$

(2) Watch out for powers of negative numbers. For example,

$$(-2)^3 = -8 \text{ and } (-2)^4 = 16, \text{ so } (-x)^5 = -x^5 \text{ and } (-x)^6 = x^6.$$

(3) In general $(ab)^n = a^n b^n$. For example,

$$(3x^2y)^3 = 3^3(x^2)^3y^3 = 27x^6y^3.$$

Exercises

1. Simplify the following expressions, leaving only positive indices in the answer.

- | | | |
|--------------------------------------|-------------------------------------------|---------------------------------------|
| (a) $4^2 \times 4^{-3}$ | (b) $\frac{3^2(2^2)^{-2}}{2^3}$ | (c) x^5x^8 |
| (d) $(y^4)^6$ | (e) $(-3)^3$ | (f) $(4ab^2c)^3$ |
| (g) $x^2z^{-3} \times (xz^2)^2$ | (h) $2^n \times (2^{-n})^3 \times 2^{2n}$ | (i) $3^m \times 27^m \times 9^{-m}$ |
| (j) $(a^{\frac{1}{2}} \times a)^5$ | (k) $\frac{(-2ab)^2}{2b}$ | (l) $\frac{(-a^4b)^3(ab)^5}{-a^8b^8}$ |
| (m) $\frac{x^{-1}y^4}{x^{-5}y^{-3}}$ | (n) $\frac{10a^3b^{-2}}{5a^{-1}b^2}^{-1}$ | (o) $x^{\sqrt{}}$ |

$$\begin{aligned}\frac{\sqrt{3}}{\sqrt{2}} + \frac{2}{\sqrt{6}} &= \frac{\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{2}} + \frac{2}{\sqrt{6}} \\ &= \frac{3}{\sqrt{6}} + \frac{2}{\sqrt{6}} \\ &= \frac{5}{\sqrt{6}} \\ &= \frac{5}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{5\sqrt{6}}{6}\end{aligned}$$

Answers to Exercises

1. (a) $\frac{1}{4}$ (b) $\frac{9}{27} = \frac{9}{127}$ (c) x^{13} (d) y^{24} (e) -27
(f) $64a^3b^6c^3$ (g) x^4z (h) 1 (i) 3^{2m} (j) $a^{15/2}$
(k) $2a^2b$ (l) a^9 (m) x^4y^7 (n) $\frac{1}{2}a^{-4}b^4$ (o) $x^{4/3}$
(p) a^5 (q) $2x^{-1/2}$ (r) a^{-2} (s) 8 (t) $\frac{8}{125}$
(u) 2

2. (a) $5\sqrt{2}$ (b) $6\sqrt{2}$ (c) $5\sqrt{3}$ (d) $\frac{2\sqrt{5} - \sqrt{10}}{10}$
(e) $\frac{5\sqrt{6}}{6}$ (f) $\frac{5\sqrt{3} - 6\sqrt{5}}{15}$